

Contrasting Quantum Cosmologies

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Abstract

We compare the recent loop quantum cosmology approach of Bojowald and co-workers with earlier quantum cosmological schemes. Because the weak-energy condition can now be violated at short distances, and not necessarily with a high energy density, a bounce from an earlier collapsing phase might easier be implemented. However, this approach could render flat space unstable to rapid expansion or baby universe production; unless a Machian style principle can be invoked. It also seems to require a flipping in the arrow of time, and violates notions of unitarity, on passing through the bounce. Preventing rapid oscillations in the wavefunction seems incompatible with more general scalar-tensor gravity theories or other classically accelerating solutions.

Other approaches such as “creation from nothing” or from some quiescent state, static or time machine, are also assessed on grounds of naturalness and fine tuning.

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1.0 Introduction

In recent work Bojowald and co-workers have applied notions of loop quantum gravity to give an alternative quantum cosmological description of the early universe - reviewed in [1]. This stems from the idea that space comes in discrete packets i.e. “granules” or “cells”. Such discrete quantum geometries can evolve via Spin networks to smooth classical spaces - for general review see [2].

At first sight such a scheme does not appear fruitful for cosmology since, in particular, flat or open Friedmann-Robertson-Walker (FRW) models are infinite from the start, so the granularity is never actually apparent. One can make a cut to enclose the big bang within a finite volume as time $t \rightarrow 0$, but then the matter or energy momentum becomes infinite within such a domain [3,4]. By making a compactification, at a scale L , this infinity can be alleviated to some extent. But still the matter will diverge as the universe is evolved back towards the initial singularity: likewise for closed FRW models. Incidentally compactifying in this way introduces possible vacuum polarization and Casimir like effects, see e.g.[5,6]. Typically the energy density ρ becomes negative so violating the weak-energy condition. For a toroidal model, one typically obtains a Casimir term, e.g.[5]

$$\rho = \langle T_0^0 \rangle = -\frac{\alpha}{L^4 a^4} \quad (1)$$

with a the scale factor and the constant α depending on the nature and number of matter fields present. More elaborate twisted scalar fields can also be possible [7,5,6], which could be detrimental if they are to drive chaotic style inflation [8].

Interestingly, loop quantum gravity is also said to alter the matter component such that the weak-energy condition is effectively violated at short distances when the granularity of space becomes significant. Recall that usually matter is diluted for an expanding universe or remains constant for an exact de Sitter solution. Likewise for the Casimir type component above. But if the weak-energy condition is violated, together with a positive energy density, it can instead grow even with expansion. Such an example was called a “whimper expanding to a big bang” [9] or if relevant to the universe today, re-dubbed “a big rip” [10].

By allowing the energy density to grow there need not be a divergence in the energy density as the universe is evolved back in time. One possible

advantage is that now a bounce from an earlier collapsing phase might be implemented close to the Planck length scale. Usually one needs to bounce before the Planck energy density is surpassed, and also prevent ordinary matter from dominating.

This bounce case is apparently suggested in Bojowald’s loop quantum cosmology approach [1]. However this seem to have a number of worrying if not serious consequences. We also contrast this with more standard ideas in quantum cosmology such as the Hartle-Hawking “no boundary” proposal [11], “creation from nothing” [12,13] or by starting from a possible quiescent state. By concentrating on conceptual issues, without too much technical distraction, we hope to see the strengths and weaknesses of the various schemes. We suggest that for this formulism of loop quantum cosmology, without a further type of Mach’s principle, the cells have apparently “free rein” and are not sufficiently constrained by the global properties of the universe. We shall exploit this weakness to show possible instability to runaway expansion or baby universe creation.

2.0 Loop cosmology and super-inflation

Bojowald and co-workers have suggested an inflationary mechanism can be driven due to the discreteness of spacetime in loop quantum cosmology [14,15]. Such a mechanism violates the weak-energy condition and so is more extreme than standard inflation. This so-called super-inflation is actually of pole-law form which has a number of unsatisfactory properties [16].

1) There is a second rapidly collapsing deflationary solution that occurs to the future of a curvature singularity. You might object that standard de Sitter also has a collapsing branch but that is to the past of the expansionary phase and can be bypassed, or matched to an earlier non-inflationary phase. But here such a deflationary solution has to be avoided and could intervene during the collapsing phase preceding a classical singularity cf.[17]. In ref.[18] they worked with this deflationary solution to produce a first collapsing before later expanding cosmological model.

2) Such a pole-law inflation has a corresponding growing Hubble parameter that produces a blue spectrum of perturbations [19]. Because the size of gravitational waves is given by the Hubble parameter it should not be allowed to become larger than $\sim 10^{-5}m_{pl}$ [20].

3) The definition of inflation is given as the ratio of final to initial scale factor a_f/a_i [14,15]. However there is another requirement that the universe be sufficiently large and produce mass density $\sim 10^{-30}gcm^{-3}$ today [21]. This

requires that a_f be $\sim cm$ size, so requiring the parameter j [1,14,15] to be extremely large.

4) It is suggested that even if the pole-law inflation is not itself sufficient, a second potential driven inflation could be produced [15,22]. This is because the friction term in the Klein-Gordon equation can change sign and drive the field up a scalar potential. However, if the kinetic term is always given by the expression $\dot{\phi} \propto a^{12}$ (cf. eq. (10) in ref. [22]) one can show that the potential term dominates over the friction term and any growth in ϕ is negligibly small. But instead of using this expression to determine the initial $\dot{\phi}$ the authors of ref.[22] use some argument based on quantum uncertainty to give a correspondingly larger initial $\dot{\phi}$: this then does allow the friction term to dominate. In my view their initial value of $\dot{\phi} \sim 10^{-5}$ is actually not “small” and for the relevant parameters a value of $\sim 10^{-22}$ would not be unwarranted. I wish to satisfy the various semi-classical equations from the start which seems more reliable than starting with “off-shell” values. So it is not yet entirely clear just how robust such a scheme is for driving the initial field up its potential hill.

Incidentally, in Euclidean space the friction term is also switched in the corresponding Klein-Gordon equation. This “anti-friction” mechanism has been used previously in conventional quantum cosmology to explain a large initial field.

2.1 Is Loop quantum cosmology unstable?

For a massless scalar field the energy density $\rho \propto a^{-6}$ is now modified at short distance such that $\rho \propto a^n$ with $n > 1$ [1,14,15,23]. At larger scales determined by the parameter j the energy density regains its standard behaviour. The energy density now disappears as $a \rightarrow 0$ and so is actually indistinguishable from flat space. But this suggests a danger that actually any Planck sized region is now potentially unstable to this inflationary expansion. One might try and reason that for a Planck length region to inflate it requires a negative pressure that will be quickly equalized by the greater average pressure of the universe outside. This was one of the reasons that creating a “universe in the lab” is difficult because of a pre-existing background metric [24]. But while such an equalization is taking place there is the possibility of a quantum tunnelling occurring to a new baby universe. This does not supplant the original universe but disconnects forming a new universe. In standard potential driven inflation such a scheme requires one to produce a high energy density false vacuum that then has a minuscule chance

of tunnelling to produce a new universe [25]. But now any Planck size region automatically could make such a transition providing topology changes are not forbidden on other grounds, see e.g.[26] for introduction to topology issues. In the “lab” it required huge effort to violate the strong-energy condition, but now the weak-energy condition is continually being violated at short distances. One might try and quantitatively calculate this enhancement but there is another ambiguity: Bojowald has introduced an arbitrary compactification scale for flat or open cosmological models. This seems a common occurrence in loop cosmology e.g. [27]. Usually, these cases have infinite action due to infinite size and are discounted, see e.g. [28]. But now with a finite volume V and energy density decreasing with size such universes are not apparently suppressed on action principles alone, $S = V \int a^n dt \rightarrow 0$ as $a \rightarrow 0$. Again in the language of ref.[9] a “whimper can expand to a big bang”. In the closed case a forbidden region is present so that the created universe must start with at least a certain size cf.[15]. Placing this value beyond the weak-energy violating region might help suppress the universe creation effect but this would introduce fine tuning.

We can also consider the creation of the original universe *ex nihilo*. Now even in standard quantum cosmology it isn’t clear why universes are not still being created around us. You can try and argue that the forbidden region creates a barrier that to observers within the existing universe suppresses further universe creation e.g.[29]. But this barrier is either absent or reduced in loop quantum cosmology and as we have argued might make universes “too easily” produced. Admittedly, loop quantum cosmology has rather suggested that the universe evolves not from “nothing” but from a previously collapsing phase [1,17]. A classical bounce is anyway possible if the strong, or for the more general case, weak-energy condition is violated, see e.g.[30]. We have already reviewed some problems that classical bouncing universes have, such as entropy production during the collapsing phase due to perturbation growth [31]. The way that this is apparently overcome with loop quantum cosmology appears a contradiction with the generalized 2nd law of thermodynamics [32] and also unitarity issues of quantum mechanics, see e.g.[33]. Instead of the energy density growing on approaching the singularity it rather decreases so that degrees of freedom are being removed: so the arrow of time is actually reversed during the collapse. This goes against notions in black hole physics that information should not actually be destroyed. We previously criticized the cyclic ekpyrotic universe model [34] for requiring a similar

entropy reduction scheme before the universe could be reset: a low-energy driven inflation alone does not alone achieve this task [31].

Even if a bounce does proceed correctly one is still left with understanding some earlier “initial state” even if this is now at a large classical scale. It might be possible to use the Hartle-Hawking type boundary condition to give an initially large universe [35], but this assumes Euclidean spacetime can also be allowed disconcertingly up to arbitrary large size [36]. It still would not apparently explain why entropy should reduce during the ensuing collapsing phase or remain negligibly small during the bouncing period.

2.2 Loop quantum avoidance of singularity?

We now wish to compare some further aspects of loop quantum cosmology with the standard approach. As an archetypal example first consider the massless scalar field in a closed FRW universe, given by the Wheeler-DeWitt (WDW) equation, e.g.[37]. We follow our earlier presentation of this example [38],

$$\left(\frac{\partial}{\partial a^2} + \frac{p}{a} \frac{\partial}{\partial a} - \frac{1}{a^2} \frac{\partial}{\partial \phi^2} - a^2 \right) \Psi(a, \phi) = 0 \quad (2)$$

where p represents part of the factor ordering ambiguity.

The WDW equation can be separated to

$$\left(a^2 \frac{d^2}{da^2} + pa \frac{d}{da} + \nu^2 - a^4 \right) \Psi(a) = 0 \quad (3)$$

$$\left(\frac{d^2}{d\phi^2} + \nu^2 \right) \Psi(\phi) = 0 \quad (4)$$

with ν the separation constant.

The solution to these equations can be obtained using MAPLE [39],

$$\Psi(a) \sim a^{(1-p)/2} \left\{ \alpha J_{i\nu/2}(ia^2/2) + \beta Y_{i\nu/2}(ia^2/2) \right\} \quad (5)$$

$$\Psi(\phi) \sim \exp(i\nu\phi) \quad (6)$$

where J and Y are Bessel functions (see e.g. [40]) and each term has an associated arbitrary constant α, β , which we can choose accordingly.

First consider the limit $a \rightarrow 0$. Using the asymptote $J_\mu(z) \sim z^\mu$ as $z \rightarrow 0$ enables the solution to be expressed as

$$\Psi(a) \sim a^{(1-p)/2} \exp(i\nu \ln a) \quad (7)$$

There is a divergence as $a \rightarrow 0$ producing an infinite oscillation representing the classical singularity as the kinetic energy of the scalar field diverges. A similar divergence also occurs for $\phi \rightarrow \infty$. In loop quantum cosmology this divergence is regulated as the discreteness of space alters the effective density [1].

But even as it stands, the solution can anyway be regularized by integrating over the arbitrary separation constant. Now the integral

$$\Psi(a, \phi) \equiv \Psi(a)\Psi(\phi) \sim \int \exp(i\nu[\ln a + \phi]) d\nu \quad (8)$$

is of the form $\int \exp(ixt)dt$ which by means of the Riemann-Lebesgue Lemma tends to zero as $x \rightarrow \infty$ (see eg. ref.[41]). The wavefunction is now damped as $a \rightarrow 0$ or $\phi \rightarrow \infty$. There is another possible divergence for factor ordering $p > 1$ but we have assumed its coordinate invariant value of unity [42]. It has also been suggested that in the context of wormhole solutions these milder divergences due to the factor ordering are not particularly serious [43]. They are also present for flat empty space, so they conceivably anyway should be renormalized away. For large scale factor the wavefunction eq.(8), behaves as $\sim \exp(-a^2/2)$ so indicating asymptotically Euclidean space.¹

Although loop quantum cosmology can also deal with the divergence of the scale factor we do not see how it necessarily can deal with the ϕ variable divergence. So although the $\Psi(a)$ part of the solution is suitably regularized the actual solution $\Psi(a, \phi)$ still can oscillate at arbitrary short “pitch”, due to the matter component. Only simple “on shell” perfect fluid models allow the matter to be expressed in terms of the scale factor. For the FRW case the scalar field has an extra degree of freedom over a perfect fluid, cf.[44].

To see this more explicitly consider the Brans-Dicke model which is derived from the following action

$$S = \int d^4x \sqrt{g} \left(\phi R - \frac{\omega}{\phi} (\partial_\mu \phi)^2 \right) . \quad (9)$$

For stability in Lorentzian space one requires $\omega > -3/2$.

Using standard techniques, the corresponding WDW equation can be

¹Provided the combination $J + iY$ which equals the first Hankel function $H^{(1)}$ [40] is chosen.

obtained [38],

$$\left(a^2 \frac{\partial}{\partial a^2} + ap \frac{\partial}{\partial a} - \frac{\phi^2}{3+2\omega} \left\{ \frac{\partial}{\partial \phi^2} + \frac{q}{\phi} \frac{\partial}{\partial \phi} \right\} - a^4\right) \Psi(a, \phi) = 0 \quad (10)$$

where p and q represent part of the now two-factor ordering ambiguities.

The WDW equation again can be separated but only the equation for $\Psi(\phi)$ differs from the previous case

$$\left(\phi^2 \frac{d^2}{d\phi^2} + q\phi \frac{d}{d\phi} + (3+2\omega)\nu^2\right) \Psi(\phi) = 0 \quad (11)$$

with ν the separation constant.

The solution is given by

$$\Psi(\phi) \sim \exp(i\sqrt{B}\nu \ln \phi) \quad (12)$$

for $q = 1$ and $B = (3+2\omega)$. So the oscillatory divergence now occurs for $\phi \rightarrow 0$ as well. Again one can integrate over the separation constant to produce a regular wave function

$$\Psi(a, \phi) \equiv \Psi(a)\Psi(\phi) \sim \int \exp(i\nu[\ln a + \ln \phi]) d\nu \quad (13)$$

Since in the limit $\phi \rightarrow 0$ the Planck length $l_p = G^{1/2} \rightarrow \infty$, you might expect loop quantum cosmology to also regulate this divergence. But in the opposite limit $\phi \rightarrow \infty$ there is also a divergence now at large distance beyond the Planck length. Although less severe than the previous divergence the “pitch” can still develop at arbitrarily short distance. We expect also rapid oscillatory wavefunctions more generally for scalar-tensor gravity models, including non-minimally coupled scalar fields cf. [45]. Also higher order correction to the gravitational action typically correspond to additional scalar fields in the Einstein frame, see e.g.[46]

One can see a similar behaviour in the WDW solution for a pure cosmological constant. The pitch of the solution gets increasingly shorter at large distance a - see for example Fig.(4) in ref.[11]. This is simply because the universe keeps accelerating and so the “velocity” $\dot{a} \rightarrow \infty$. We have remarked [31] that if loop quantum cosmology regulated such solutions it could prevent eternal inflation to the future. However, now this singularity avoidance

mechanism might over-constrain such models and prevent wanted solutions. Bojowald mentions [47] that such examples are merely “infrared problems” and can be ignored since the local curvature is still small. But this distinction seems arbitrary, especially since the loop approach matches to flat space in the early universe, and not just when large energy densities are present.

3.0 Standard Quantum Cosmology

We have described how the massless scalar field example can be regulated by integration over the separation constant. However, in the common boundary conditions such as “no boundary” or tunnelling ones this constant is simply zero, see e.g.[48]. So by fiat the matter is expunged and any possible oscillatory divergences are simply absent. This allows any inflationary matter present to become dominant during the early stage of the universe, so avoiding ambiguities found in classical measures for inflation [49]. One might question whether this imposition is reasonable and indeed it has been suggested that “zero point” fluctuations would alone alter this picture [50].² But it does remove any potential singularity due to stiff matter, contrary to the impression of Bojowald that singularities couldn’t be removed in standard quantum cosmology cf.[17]. It is true however, that the presence of strong-energy violating matter is a strict requirement for these common boundary conditions otherwise no natural Euclidean or forbidden region would be present.

The presence of a forbidden region allows both exponentially growing and decaying solutions so there can be big differences in prediction, see e.g.[37]. The action of a de Sitter solution driven by a scalar potential $V(\phi)$ is negative $\sim -1/V(\phi)$ [51] and whether one should first take the modulus alters the corresponding tunnelling rate $\sim \exp(-\text{Action})$ behaviour. There is some dispute whether the Hartle-Hawking case $\sim \exp(1/V(\phi))$ gives sufficient inflation [52]. This depends on whether you can include energy densities beyond the Planck value.

We have earlier questioned whether this analogy of treating the universe like α decay or a Scanning Tunnelling microscope is sensible [53]. In these atomic examples the various particles already exist. But now the “particle” is the universe itself coming into existence. Another point is that if the barrier is removed you would expect to get a stream of particles from the

²One could make a similar argument against loop quantum cosmology now concerning the apparent absence of Casimir term, or vacuum polarization, as $a \rightarrow 0$

reservoir. This barrier can be removed in flat or open cosmological models but is replaced by the infinite action of any matter filling a now infinite universe [28]. But if we compactify the space at arbitrary small size we can reduce the action to an arbitrary small amount cf. [54]. In this case the “reservoir of universes” can drain away. We have suggested that the loop quantum cosmology approach is especially susceptible to this dilemma but it could also be taken as a *reductio ad absurdum* to standard quantum cosmology as well. Although the loop case also seemingly allows baby universes to easily break off from a pre-existing universes.

There is also the issue of including different topologies and geometries for the tunnelling amplitude in the more general case. Because the number of manifolds for the hyperbolic case can approach infinity it can overwhelm the usual suppression factor for the creation of a single universe with a set topology [55]. The “average” topology might be able to predict the spatial homogeneity of the universe [56]. But again is this really reasonable? It implies an initial state or “reservoir” of all infinite possible topologies that should be included in the amplitude. There are now infinitely many “particles” one for each possible topology and geometry. Neither is it clear why just a single universe with an average topology results and not that many universes each with different topology form together. Working with closed models, and so fewer possible topologies, see e.g. [57], could alleviate this problem but the notion of curvature itself will anyway become hazy at the Planck scale.

Indeed the way that curvature is treated as a constant is rather unsatisfactory. In FRW models the actually local characteristic k is taken to be globally constant. In more general metrics the curvature can become a function also of time and space $k(t, x)$ cf. Stephani models e.g.[58]. For the FRW model with perfect fluid $p = (\gamma - 1)\rho$ the WDW potential takes the form, e.g.[44]

$$U = ka^2 - Aa^{4-3\gamma} \quad (14)$$

where the constant A can be obtained from the relation $\rho = A/a^{3\gamma}$. For a forbidden or Euclidean region at small scale factor a requires $U > 0$ which requires $k = 1$ and violation of the strong-energy condition i.e. $0 \leq \gamma < 2/3$. However, in a more general inhomogeneous model this behaviour can be drastically altered. For example in a Stephani model the corresponding

WDW potential becomes cf. [59]

$$U = \beta a^n - Aa^{4-3\gamma} \quad (15)$$

so for $n > 2$ the forbidden region can be either narrowed or absent entirely even for closed models $\beta > 0$ and when the strong-energy condition is being violated. This example is symptomatic of what, more realistically, can be expected as the Planck epoch is approached. The presence of forbidden regions that play such a prominent role might not then actually be present, even in closed models undergoing inflation. A slight complication is that a negative energy density can also create a forbidden region. For example a flat toroidal universe has a Casimir term corresponding to a $\beta > 0$ and $n = 0$ term in eq. (15) [13].

Even without a forbidden region some boundary conditions might be adapted to purely Lorentzian metrics, although the underlying principle is then often less prescriptive cf.[54]- where the “outgoing only ” aspect of the Tunnelling boundary condition was implemented in such cases.

4.0 Universe from a quiescent or static state

We have spoken of the universe starting from nothing or by bouncing from a previously collapsing phase. A third possibility is that originally the universe was initially stuck in some unchanging quiescent state. Perhaps involving the presence of closed timelike curves (CTCs), see e.g.[26] for review. This is closely related to introducing a topological identification scale as in Misner space.

Starting with Misner space, Gott and Li [50,60] obtained a self-consistent adapted Rindler vacuum state for a conformally coupled scalar field that remains finite at the Cauchy horizon, unlike for the Minkowski case [61]. They then conformally transformed this state to give a suitable vacuum state for multiply connected de Sitter space. Such a de Sitter space with CTCs could be a suitable initial state for the universe. It only has retarded solutions so giving an “arrow of time” and is a state of low entropy, actually of zero temperature [50] .

However, in Misner space this state was only possible with identification scale $b = 2\pi$, or $b = 2\pi r_0$ for the multiple de Sitter case [50,60] . Such an exact value is itself inconsistent with notions of quantum uncertainty. We are wary of claims that such a multiply connected de Sitter state is stable especially since the relevant time loop is approximately \sim Planck time, only a plausibility argument has so far been made [62].

The actual procedure of balancing a negative starting vacuum with a Hawking radiation due to the periodicity to give an empty vacuum state has possible difficulties. The calculation makes use of the periodicity producing a thermal state [63]. Such a state is required to be a many particle state with technically a suitably large Fock space, see e.g.[6]. But by being close to the Planck scale one starts reducing the number of allowed states due to holography type arguments [64]. This will start preventing an exact thermal state, as also is expected during the final stages of black hole evaporation [65] or in Planck scale de Sitter space [51]. This mismatch could then result in some fluctuations still being present in the vacuum instead of a pure empty state, so destabilizing the CTC.

Neither is it clear that the b value, or the corresponding de Sitter one, remain independent of different matter couplings ξ or potentials $V(\phi)$. A more realistic combination of matter sources still appears divergent at the Cauchy horizon [66], although an improved *self-consistent* renormalization procedure [67] in Euclidean space might help regulate some of these other cases.

Creating this state in any case seems rather contrived. Recall that the Rindler vacuum of accelerating observers requires “mirrors and absorbing stray radiation”, before we then make any topological identification [68]. One would need some more general reason why such an initial state was actually present. The analogous zero temperature state for charged Black holes has proved difficult to obtain on grounds of stability [69].

Instead of requiring CTCs one might just allow a static state with time still evolving normally from say $-\infty$. There is a recent emergent model [70], an update of the Eddington-Lemaître model [9] that starts from an Einstein static universe. Because this model has no forbidden region, and requires a balance of ordinary matter and cosmological constant, it again will be prevented by the usual boundary conditions that bias against the normal e.g. radiation matter component. Neither do we think that maximizing the entropy is a more suitable boundary condition since the entropy actually grows later during the inflationary stage cf.[70]. The emergent model is however geodesically complete to the past unlike the previous case with multiply connected de Sitter space.

Also such a model also requires a mechanism to stabilize the Einstein static phase to homogeneous perturbations. More general inhomogeneous models might allow this. For example, by altering the curvature dependence

as in eq.(15) one could produce a stable static universe with a now flat $U = 0$ WDW potential; or perhaps, at least prevent collapse to the origin by means of a repelling potential $U \gg 0$ around the origin $a = 0$, cf.[48].

One might also try to stabilize the Einstein static universe more generally by surrounding the state entirely with forbidden or Euclidean regions. For example if the sign of the WDW potential U is flipped the corresponding Einstein static universe is stabilized. Such a model requires $k; \Lambda; \rho; \rightarrow -k; -\Lambda; -\rho$, so now this is an open Anti-de Sitter with negative radiation model. So violation of the weak-energy condition is now required for such stability, such as might occur in the Casimir effect cf. eq.(1). This could also be achieved without altering the matter component by use of a signature change, represented by the parameter ϵ : $\epsilon = 1$ for usual Lorentzian space and -1 for Euclidean space [71]. In the simplest case the corresponding WDW potential is altered $U \rightarrow \epsilon U$ [44]. So if for some reason $\epsilon = -1$ the previous static universe is stabilized. Other possible examples starting from different action principles are also possible [72].

It has been suggested that oscillations of ϵ between the two cases are constantly occurring but that the “average” is now in the Lorentzian region [73]. One might imagine instead a preponderance of negative Euclidean values for ϵ . This might help stabilize a static model before for some reason the sign changed and Lorentzian evolution then could proceed.

Despite these present difficulties the notion of finding a suitable quiescent state has some attraction. The difficulty, as in the examples given, is why the state should survive for semi-infinite times, but still have some slight instability that causes the expansionary evolution to begin.

5.0 Conclusions

Although most work on quantum gravity is still being done within string theory, loop quantum gravity is also making progress. One advantage of loop quantum gravity over strings is that a background independent formalism might easier be achieved [2]. One drawback however is that in GUT theories the various forces of nature should eventually unify. Therefore the present weak force of gravity should increase with energy scale to eventually coincide with the other forces of nature. The Planck length $G^{1/2}$ will correspondingly grow as the unification is achieved. But in loop gravity this aspect of “running” Planck length is apparently not incorporated at present. Incidentally, such a large initial Planck scale could alter predictions for the initial state and any subsequent requirements of inflation.

The idea of space being made of discrete quanta might introduce further conceptual problems. In an expanding model new cells have to be produced to fill in the gaps. But if we make analogy with cell division in living organisms, how are cells produced without error? Because presumably there is no analogy with DNA, there seems the need of providing “scaffolding” to force cells to have their correct form. One might try and claim the classical equations impose this by stricture, but if only a few cells are present the classical structure is still unformed. Constant quantum fluctuations at short distance have continually now to be kept in check.

Because loop quantum cosmology allows the weak-energy condition to be violated it allows more variety than with more standard matter sources. Of course one might allow such phantom matter in standard quantum cosmology but then there is no apparent scale where the effect can be turned off: the growing energy density will also eventually grow beyond Planck values that we have some confidence in describing. However, such properties are in danger of making flat space itself unstable to expansion, or to baby universe production. Bojowald has suggested to me that, in the above language, there now exists a scaffolding preventing such exotic behaviour due to the universe now obeying the “average” classical description. We are suspicious how individual cells know about the average and so behave appropriately. Of course in the formalism this is innocuously hidden in the scale factor, which plays this “non-local” messenger role. In standard cosmology space is never created in this way but is simply stretched like an elastic band by the scale factor. Even then gravitationally bound systems e.g. galaxies can drop out of the expansionary global behaviour of the universe: so the scale factor never plays a universal messenger role to individual atoms. We seem instead, for loop cosmology, in need of a sort of “generalized Mach’s principle” e.g. [4,9], telling the individual granules how the universe actually is on average. A related concern [74] seems present in certain variable constant theories (see e.g. [75]), that also use the scale factor to determine, for example, the varying speed of light. Maybe this adapted Mach’s principle only needs to work up to some suitable classical size, a meter or so. Otherwise without such a constraint the instabilities I have suggested seem viable.

The general idea that short distance oscillations in the wavefunction should be excluded due to granular effects might at first sight appear to be reasonable. However, such oscillatory behaviour is not readily confined to short distance in the scale factor *per se*. We can also question how Einstein’s

equations $G_{\mu\nu} \Leftrightarrow T_{\mu\nu}$ have been used to interchange between matter and geometry i.e. $\phi \leftrightarrow a$. Ideally this “on shell” relationship should be quantized and then not be used again. Care is also required if such relationships are used to remove degrees of freedom, that although irrelevant classically, can alter the full quantum theory cf.[44].

In more general scalar-tensor gravity, or with higher order corrections to the gravitational action, this distinction between the geometry and matter is even more mixed up. The total solution can have arbitrary oscillations that cannot easily be confined or excluded by discreteness in the scale factor alone. Either this extra scalar should be excluded or is somehow itself unaffected by the discreteness of space. Other standard models such as with a simple cosmological constant also get arbitrary short oscillation lengths corresponding to increasing kinetic energy. We have therefore suggested that this property of quantum gravity becoming important at short-distance is too general. One should require that also the energy-density be approaching large or even Planck values.

The notion that a bounce could occur close to the Planck distance also seems to have unforeseen consequences. Because matter is actually being diluted, even during contraction, the corresponding arrow of time always points away from the bounce point. Neither is this consistent with unitarity: that information should not readily be destroyed. Not only actual matter but the vacuum state itself must be adjusted. Otherwise, vacuum polarization effects (like Casimir and Conformal anomaly) would be expected to dominate cf.[76].

We have contrasted various common approaches to quantum cosmology and admittedly none of them are without conceptual difficulties. Of course some topics, particularly involving higher dimensions, have not been considered here, but the issues are still rather universal. Incidentally we have earlier criticized the use of quantum cosmology in Brane type cosmologies on the grounds that the initial bulk space is infinite, and Branes have large action *ab initio*: neither of which is really amenable to a quantum creation description [77]. Perhaps the bounce type scheme is actually more appropriate, as attempted in the ekpyrotic universe scheme, provided the singularity is suitably regulated.

The standard quantum cosmological schemes are found to have certain *fragility* problems: like that forbidden regions are strongly dependent on how the curvature behaves, or time machines require extreme fine tuning. Anal-

gies with atomic physics, such as quantum tunnelling, are extrapolated to the universe as a whole. Usually the boundary conditions have been developed apparently with the sole aim of starting the universe in an inflationary state. One then at least must start with matter sources that could produce inflation. Although, there is still some dispute whether this is achieved, it does help prevent ambiguities in purely classical measures for the probability of inflation. However, what preceded this inflationary state, and why and how it then evolved, is far from clear. Perhaps a better explanation is still to be found within the realms of quantum gravity. Whether a single quanta of spacetime - a modern version of Lemaitre's "primeval atom" - is involved or an entirely new conceptual approach (from M theory?) certainly remains a fascinating topic for the future.

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